FLOW OF HIGHLY RAREFIED GASES AROUND OSCILLATING SURFACES

(OBTEKANIE KOLEBLIUSHCHEISIA POVERKHONOSTI POTOKOM SIL'NO RAZREZHENNOGO GAZA)

PMM Vol.23, No.4, 1959, pp. 760-765

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(Received 9 January 1959)

References [1-3] determined pressure, frontal drag, heat transfer, and skin friction of blunt bodies moving uniformly in free-molecular flow of rarefied gas.

In the present paper, the total force, acting on a unit surface, which performs small unsteady motions while moving forward, is determined, as well as its projection in directions corresponding to normal pressure, friction, etc. An attempt is made to assess the validity of analysis of flows around concave surfaces with the usual free-molecular assumptions. In the process, conditions on the shape of such surfaces and on their motion are established. As a concrete example, the flow around an oscillating flat plate is analyzed. It is found that the expression for the additional pressure due to the oscillation agrees, up to a multiplicative constant, with a known formula of the "piston theory", which can well be of theoretical interest.

1. Let the surface move forward through the rarefied gas with velocity **v** relative to a coordinate system* fixed in space and designate this motion as "undisturbed". Also let the body perform small oscillations relative to this undisturbed motion. We shall take the molecular distribution in the approaching stream to be Maxwellian. The distribution function referred to a local coordinate system embedded in the surface, takes the form

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* Translator's Note: The reader will find it helpful to refer to Section 2, following equation (2.2), for specific definitions of the fixed and local coordinate systems used by the author.

$$f(\mathbf{c}, \mathbf{u}) = \begin{cases} N_\infty (h_\infty / \pi)^{3/2} \exp[-h_\infty (\mathbf{c}+\mathbf{u})^2] & \text{when } \mathbf{c}.\mathbf{n} < 0 \\ N_\infty (h_\infty / \pi)^{3/2} (1-\varepsilon) \exp\{-h_\infty [\mathbf{c}+\mathbf{u}-2\mathbf{n}(\mathbf{c}.\mathbf{n})]^2\} + \\ \quad + \varepsilon N (h/\pi)^{3/2} \exp\{-h\mathbf{c}^2\} & \text{when } \mathbf{c}.\mathbf{n} > 0 \end{cases} \tag{1.1}$$

$$(h = m_0 / 2kT)$$

Here **c** is the molecular velocity relative to the surface, **u** the velocity of disturbed motion of the surface relative to the fixed reference system, **n** - the normal to the surface in its perturbed instantaneous position, N - number of molecules per unit volume, m_0 - the mass of individual molecules, k - the Boltzman constant. In accordance with [1], the expression for T was taken in the form* $T = T_0 + a/\epsilon\,(T_w - T_0)$ where T_w is the temperature of the wall, a - the accommodation coefficient, T_0 - the throttling temperature. The subscript refers to parameters of the medium at infinity, while the absence of subscripts indicates conditions of the medium near the body.

The distribution function (1.1) corresponds to free-molecular flow around the body when the mean free path exceeds the characteristic length of the body. For the model of interaction between molecules and surface a combination of specular and diffuse reflection [1, p. 662] in the ratio ϵ is assumed.

We determine the force acting on the surface from the relation

$$d\mathbf{K} = -\mathbf{F}\,dt$$

K is the impulse. From the change in **K** due to the impact of molecules on the surface we find the force per unit time:

$$\mathbf{F} = -\left\{m_0 \int_{-\infty}^{\infty} dc^1 \int_{-\infty}^{\infty} dc^2 \int_0^{\infty} (\mathbf{cn})\,\mathbf{c} f(\mathbf{c}, \mathbf{u})\,dc^3 + m_0 \int_{-\infty}^{\infty} dc^1 \int_{-\infty}^{\infty} dc^2 \int_{-\infty}^{0} (\mathbf{cn})\,\mathbf{c} f(\mathbf{c}, \mathbf{u})\,dc^3\right\}$$

Substituting for $f(c, u)$ from expression (1.1) we obtain:

$$\mathbf{F} = -\varepsilon\rho_\infty \mathbf{u} \left\{\frac{\mathbf{u}\cdot\mathbf{n}}{2}[1 + \operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{2\sqrt{\pi h_\infty}}\right\} -$$

$$-\rho_\infty(2-2\varepsilon)\,\mathbf{n}\,(\mathbf{u}\cdot\mathbf{n})\left\{\frac{\mathbf{u}\cdot\mathbf{n}}{2}[1 + \operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{2\sqrt{\pi h_\infty}}\right\} -$$

$$-\frac{(2-\varepsilon)\,\rho_\infty \mathbf{n}}{4h_\infty}[1 + \operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] - \frac{\varepsilon\rho\mathbf{n}}{4h} \tag{1.2}$$

Projection of the vector **F** on − **n** (with minus sign because the positive direction of the normal is outward) leads to the expression for the pressure

* Translator's Note: ϵ is the fraction of impinging molecules which are reflected diffusely.

$$p = (2-\varepsilon)\rho_\infty(\mathbf{u}\cdot\mathbf{n})\left\{\frac{\mathbf{u}\cdot\mathbf{n}}{2}[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{2\sqrt{\pi h_\infty}}\right\} + \frac{(2-\varepsilon)}{4h_\infty}\rho_\infty[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{\rho\varepsilon}{4h} \quad (1.3)$$

Projecting $\mathbf{F}$ in the tangential direction, $\mathbf{t}$, in the plane of the local normal and the velocity $\mathbf{u}$, we find the expression for frictional stress:

$$\tau = -\varepsilon\rho_\infty(\mathbf{u}\cdot\mathbf{t})\left\{\frac{\mathbf{u}\cdot\mathbf{n}}{2}[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{2\sqrt{\pi h_\infty}}\right\} \quad (1.4)$$

Similarly one can evaluate frontal drag and total lift.

Following the same procedure one can derive the expression for the amount of energy, E, which the impinging molecules transfer to the surface per unit time. (Here only the kinetic energy of the molecules is taken into account.)

$$E = -\frac{m_0}{2}\int_{-\infty}^{\infty}dc^1\int_{-\infty}^{\infty}dc^2\int_{-\infty}^{0}(\mathbf{c}\cdot\mathbf{n})\,\mathbf{c}^2 f(\mathbf{c},\mathbf{u})\,dc^3 - \frac{m_0}{2}\int_{-\infty}^{\infty}dc^1\int_{-\infty}^{\infty}dc^2\int_{0}^{\infty}(\mathbf{c}\cdot\mathbf{n})\,\mathbf{c}^2 f(\mathbf{c},\mathbf{u})\,dc^3$$

With expression (1.1) for $f(\mathbf{c},\mathbf{u})$ one has:

$$E = \frac{\varepsilon\rho_\infty}{2}\left\{\frac{\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{h_\infty\sqrt{\pi h_\infty}} + \frac{\mathbf{u}^2\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2]}{2\sqrt{\pi h_\infty}} + \frac{(\mathbf{u}\cdot\mathbf{n})\mathbf{u}^2}{2}[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] + \frac{5}{4}\frac{(\mathbf{u}\cdot\mathbf{n})}{h_\infty}[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})] - \frac{\rho}{2h\sqrt{\pi h}}\right\} \quad (1.5)$$

The expression $\rho = m_0 N$, which appears in equations (1.2)-(1.5), can be evaluated by considering the conservation of mass of the surface

$$-m_0\int_{-\infty}^{\infty}dc^1\int_{-\infty}^{\infty}dc^2\int_{-\infty}^{0}(\mathbf{c}\cdot\mathbf{n})f(\mathbf{c},\mathbf{u})\,dc^3 = m_0\int_{-\infty}^{\infty}dc^1\int_{-\infty}^{\infty}dc^2\int_{0}^{\infty}(\mathbf{c}\cdot\mathbf{n})f(\mathbf{c},\mathbf{u})\,dc^3$$

Hence

$$\rho = \rho_\infty\left\{\sqrt{\frac{T_\infty}{T}}\exp[-h_\infty(\mathbf{u}\cdot\mathbf{n})^2] + (\mathbf{u}\cdot\mathbf{n})\sqrt{\pi h}\,[1+\operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})]\right\} \quad (1.6)$$

The expression for E can be utilized for the determination of the heating of the body when additional specific assumptions are made about the character of heat transfer.

2. In the derivation of formulas (1.2)-(1.6) it was assumed that the impinging molecules come to the surface from "infinity" in the sense that the distribution function was taken to be Maxwellian when $(\mathbf{c}\cdot\mathbf{n}) < 0$, corresponding to "boundary conditions" which, strictly speaking, are valid only for blunt bodies. We shall assess the errors allowed in the derivation of (1.2-(1.6) by neglecting the fact that for concave bodies the locally impinging molecules not only come from "infinity" but also arrive after reflection from another part of the surface.

With reference to the figure, one sees that for rigid surfaces such reflected molecules encounter the surface again, if the angle of their path relative to the local tangent is smaller or equal to β. Since the velocities of the molecules are very large, one can assume that distances comparable to dimensions of the body in question are traversed by the molecules instantaneously. Then the counting of the molecules which meet the surface again after an earlier reflection, in the case of surfaces deforming with time, proceeds just as in the case of rigidly moving surfaces.

Let us find the number of molecules reflecting at angle β or less:

$$n_\beta = \iiint_\Omega \mathbf{c} \cdot \mathbf{n} f(\mathbf{c}, \mathbf{u})\, d\Omega$$

The domain of integration Ω is*

$$-\infty < c^1 < \infty, \qquad -\infty < c^2 < \infty, \qquad 0 \leqslant c^3 \leqslant \operatorname{tg}\beta \sqrt{(c^1)^2 + (c^2)^2}$$

Evaluating n_β, we obtain:

$$n_\beta = \frac{\varepsilon N}{2\sqrt{\pi h}} \frac{\operatorname{tg}^2\beta}{1+\operatorname{tg}^2\beta} + (1-\varepsilon) N_\infty (\mathbf{u}\cdot\mathbf{n}) \sin\beta \left\{ \sqrt{\frac{h_\infty}{\pi}}\, u \exp(-h_\infty u^2) + \right.$$
$$\left. + \tfrac{1}{4}\left[1 - \operatorname{erf}(\sqrt{h_\infty}\,\mathbf{u}\cdot\mathbf{n})\right] \right\} - \frac{N_\infty (1-\varepsilon)}{2\sqrt{\pi h_\infty}} \sin^2\beta \exp(-h_\infty u^2) -$$
$$- \tfrac{1}{4} \sin^2\beta N_\infty \sqrt{\frac{h_\infty}{\pi}}\, u^2 \exp(-h_\infty u^2) \qquad (2.1)$$

It is clear from (2.1) that to ensure that n_β be small the following conditions are sufficient:

$$\sin^2\beta \ll 1, \qquad \operatorname{tg}^2\beta \ll 1, \qquad \left(\frac{\mathbf{u}\cdot\mathbf{n}}{c^0}\right)^2 \ll 1 \qquad (2.2)$$

where c^0 is the most probable molecular speed in the undisturbed medium.

Then it is permissible to neglect the molecules which meet the surface a second time after reflection in comparison with the totality of impinging molecules. As can be seen from (2.2), the error decreases, as $u = |\mathbf{u}|$

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* Translator's Note: The axis identified by superscript 3 is in the normal direction **n**.

increases.

Let the location of the undisturbed surface be characterized by the radius vector

$$\mathbf{r}(x^i, t) = \mathbf{r}_0(x^i) + \mathbf{v}t$$

relative to the coordinate system fixed in space. Let the principal coordinate system be defined by $\mathbf{r}_1$ $\mathbf{r}_2$ $\mathbf{r}_3$ $= \nu$ where ν is the unit vector normal to the surface in the undisturbed motion. Let the position of the deforming surface be given by the radius vector

$$\mathbf{R}(x^i, t) = \mathbf{r}(x^i, t) + \mathbf{w}(x^i, t)$$

where $\mathbf{w}(x^i, t) = w^a r_a$ is the displacement vector. The velocity of the disturbed motion of the surface becomes:

$$\mathbf{u} = u^\alpha \mathbf{r}_\alpha = \frac{\partial \mathbf{R}}{\partial t} = \frac{\partial \mathbf{r}}{\partial t} + \frac{\partial \mathbf{w}}{\partial t} = (v^\alpha + w^\alpha{}_{./t})\,\mathbf{r}_\alpha$$

Evaluating $\mathbf{n} = n^a \mathbf{r}_a$ and $\mathbf{u} \cdot \mathbf{n} = u^a n_a$, one can see that, in order to make n_β small, it is sufficient to consider only small v^3/c^0, $w^3/_t c^0$, $w^3/_j$; so that only the terms linear in these quantities need be kept in the expressions for $\mathbf{n}$ and $\mathbf{u} \cdot \mathbf{n}$. From geometry (see Fig. 1) one can see that the preceding conditions are sufficient to satisfy conditions (2.2). The notation $w^i{}_{./j}$, after Kagan [4], signifies covariant differentiation, carried out relative to the local base $\mathbf{r}_i$.

Putting $w^3{}_{./3} = -1$ in order to facilitate summation, we obtain to the stipulated accuracy

$$\mathbf{n} = -\mathbf{r}_\alpha g^{\alpha\beta} w^3_{./\beta}, \qquad \mathbf{u}\cdot\mathbf{n} = w^3{}_{./t} - v^\mu w^3_{./\mu}$$

where $g^{\alpha\beta}$ is the metric tensor of the coordinates r_i.

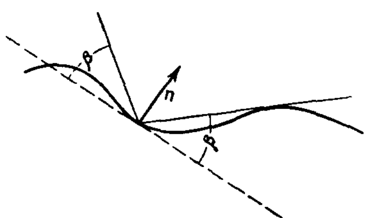

Fig. 1.

Substituting $\mathbf{n}$, $\mathbf{u}$, $\mathbf{u} \cdot \mathbf{n}$ in (1.2)-(1.6) and keeping only terms linear in the small quantities, we obtain

$$\mathbf{F} = \mathbf{r}_\alpha \left\{ -\frac{\varepsilon\rho_\infty c^0}{2\sqrt{\pi}}(v^\alpha + w^\alpha{}_{./t}) - \frac{\varepsilon\rho_\infty}{2}(v^\alpha - \delta_3{}^\alpha v^3)(w^\alpha{}_{./t} - v^\mu w^3{}_{./\mu}) + \right.$$

$$+\left[\frac{p_\infty(2-\varepsilon)}{2}+\frac{\varepsilon p_\infty}{2}\sqrt{\frac{T}{T_\infty}}\right]g^{\alpha\beta}w^3{}_{\cdot/\beta}-$$
$$-\delta_3^{\alpha}\left[\frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4}+\frac{(4-3\varepsilon)\,\rho_\infty c^\circ}{2\sqrt{\pi}}\right](w^3{}_{\cdot/t}-v^\mu w^3{}_{\cdot/\mu})\Bigg\} \qquad (2.3)$$

$$p=\left[\frac{2-\varepsilon}{2}+\frac{\varepsilon}{2}\sqrt{\frac{T}{T_\infty}}\right]p_\infty+$$
$$+\left[\frac{\rho_\infty(2-\varepsilon)c^\circ}{\sqrt{\pi}}+\frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4}\right](w^3{}_{\cdot/t}-v^\mu w^3{}_{\cdot/\mu}) \qquad (2.4)$$

$$\tau=-\frac{\varepsilon\rho_\infty}{2}\sqrt{u^\alpha u^\beta g_{\alpha\beta}}\left(\frac{c^\circ}{\sqrt{\pi}}+w^3{}_{\cdot/t}-v^\mu w^3{}_{\cdot/\mu}\right) \qquad (2.5)$$

(Here, the vector **t** is determined to the same accuracy from the conditions that $\mathbf{t}\cdot\mathbf{n}=0$, $|\mathbf{t}|=1$ and that **t**, **n** and **u** are situated in the same plane).

$$E=\frac{\varepsilon\rho_\infty}{2}\left\{\frac{2R\,c^\circ}{\sqrt{\pi}}(T_\infty-T)+\frac{c^\circ}{2\sqrt{\pi}}(v^\alpha v^\beta g_{\alpha\beta}+2g_{\alpha\beta}v^\alpha w^\beta{}_{\cdot/t})+\right.$$
$$\left.+\frac{1}{2}[v^\alpha v^\beta g_{\alpha\beta}(w^3{}_{\cdot/t}-v^\mu w^3{}_{\cdot/\mu})]+\frac{R}{2}(5RT_\infty-4T)(w^3{}_{\cdot/t}-v^\mu w^3{}_{\cdot/\mu})\right\} \qquad (2.6)$$

3. As an illustration, let us consider the case of a flat plate sliding at zero angle of attack with a velocity **v** in the direction of the x^1 axis. Here x^1 and x^2 are Cartesian coordinates in the plane of the plate. Let the oscillatory motion be restricted to the x^3, i.e. ν direction. Then

$$w^1\equiv w^2\equiv 0,\qquad v^2\equiv v^3\equiv 0,\qquad w^3=w,\qquad w^3{}_{\cdot/j}=\frac{\partial w}{\partial x^j},\qquad w^3{}_{\cdot/t}=\frac{\partial w}{\partial t}$$

In this case the forces acting on the plate can be expressed in terms of the derivatives of the plate deflections and of the thermodynamic variables

$$\mathbf{F}=\mathbf{r}_1\left\{-\varepsilon\rho_\infty v\left(\frac{\partial w}{\partial t}-v\frac{\partial w}{\partial x^1}\right)-\frac{\varepsilon\rho_\infty c^\circ}{2\sqrt{\pi}}v+\frac{\partial w}{\partial x^1}\left[\frac{(2-\varepsilon)}{2}p_\infty+\frac{\varepsilon p_\infty}{2}\sqrt{\frac{T}{T_\infty}}\right]\right\}+$$
$$+\mathbf{r}_2\frac{\partial w}{\partial x^2}\left[\frac{(2-\varepsilon)}{2}p_\infty+\frac{\varepsilon p_\infty}{2}\sqrt{\frac{T}{T_\infty}}\right]+\mathbf{r}_3\left\{\left[-\frac{(4-3\varepsilon)\rho_\infty\sqrt{2RT_\infty}}{2\sqrt{\pi}}-\right.\right.$$
$$\left.\left.-\frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4}\right]\left(\frac{\partial w}{\partial t}-v\frac{\partial w}{\partial x^1}\right)-\left[\frac{(2-\varepsilon)}{2}p_\infty+\frac{\varepsilon p_\infty}{2}\sqrt{\frac{T}{T_\infty}}\right]-\frac{\varepsilon\rho_\infty\sqrt{RT_\infty}}{\sqrt{\pi}}\frac{\partial w}{\partial t}\right\} \qquad (3.1)$$

$$p = p_\infty\left[\frac{(2-\varepsilon)}{2} + \frac{\varepsilon}{2}\sqrt{\frac{T}{T_\infty}}\right] + \left(\frac{\partial w}{\partial t} - v\frac{\partial w}{\partial x^1}\right)\left[\frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4} + \frac{\rho_\infty(2-\varepsilon)c^\circ}{\sqrt{\pi}}\right] \tag{3.2}$$

$$\tau = -\frac{\rho_\infty \varepsilon v}{2}\left[\frac{c^\circ}{\sqrt{\pi}} + \frac{\partial w}{\partial t} - v\frac{\partial w}{\partial x^1}\right] \tag{3.3}$$

The rate of energy transfer to the plate is

$$E = \frac{\varepsilon\rho_\infty}{2}\left\{2R\frac{c^\circ}{\sqrt{\pi}}(T_\infty - T) + \frac{v^2 c^\circ}{2\sqrt{\pi}} + \right.$$
$$\left. + \left[\frac{v^2}{2} + \frac{R}{2}(5T_\infty - 4T)\right]\left(\frac{\partial w}{\partial t} - v\frac{\partial w}{\partial x^1}\right)\right\} \tag{3.4}$$

Consider the expressions for the pressure (1.3) and (3.2). When the surface is not disturbed, the pressure on it is

$$p_0 = p_\infty\left[\frac{2-\varepsilon}{2} + \frac{\varepsilon}{2}\sqrt{\frac{T}{T_\infty}}\right]$$

where $p_\infty = \rho_\infty R T_\infty$, the pressure in the undisturbed medium. The pressure on the plate reduces to that in the free stream either when $\epsilon = 0$ (pure specular reflection, corresponding to the limiting case of an ideal fluid) or when $a = 0$ (no interchange of energy between gas and surface [1]). In case of interaction between molecules and surface, in which there is exchange of energy, the pressure and the density at the quiescent surface differ from p_∞ and ρ_∞ of the undisturbed medium. This possibility in rarefied gases was already noted by Maxwell [5].

Consider the additional pressure on the plate due to its disturbed motion:

$$\Delta p = p - p_0 = \left[\frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4} + \frac{\rho_\infty(2-\varepsilon)c^\circ}{\sqrt{\pi}}\right]\left(\frac{\partial w}{\partial t} - v\frac{\partial w}{\partial x^1}\right) \tag{3.5}$$

Expression (3.5) agrees up to a multiplicative constant with the linearized formula for the pressure in the "piston" theory. This agreement is natural, since the restrictions on the form of the surface and its angle of attack are the same in the two cases, and the limiting conditions on the distribution function in the form (1.1) effectively imply the hypothesis of "plane sections" on which the "piston" theory is based.*

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* Translator's Note: See Section 2.4 of Hayes-Probstein: *Hypersonic Flow Theory*, Academic Press 1959, for discussion of the hypothesis, in English.

The difference consists mainly in the effect of diffusive reflections, i.e. in allowing for partial or full adherence of the gas to the surface (governed by the magnitude of ϵ), we pass beyond the domain of ideal gases (in the sense of limiting conditions).

In a series of papers (for instance, [6]) the interaction between thin elastic panels and gas streams was considered in which the additional pressure due to the panel motion was accounted for by the piston theory. Simple conversion of magnitudes to the case of rarefied gases shows that many aeroelastic effects observed at ordinary altitudes could be significant at very high altitudes but only at high flight speeds not yet reached (forces otherwise being negligibly small). One can show, however, that in certain unfavorable cases in presence of thermal stresses, questions of dynamic stability of panels and shells may be of definite interest.

Expressions similar to (3.2)-(3.5) can be obtained for a cone moving at zero angle of attack with a velocity **v**. The cone angle must be small so that terms like $(v/c^0)\sin\beta$ can be neglected. The meridional angle ϕ and the distance l along the generators of the cone serve as the basic polar coordinates. Then, with identical assumptions on the disturbed motion of the surface:

$$\Delta p = \left[\frac{(2-\varepsilon)\rho_\infty c^\circ}{\sqrt{\pi}} + \frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4}\right]\left(\frac{\partial w}{\partial t} - v\cos\beta\frac{\partial w}{\partial l} + v\sin\beta\right)$$

$$\tau = -\frac{\varepsilon\rho_\infty v\cos\beta}{2}\left[\frac{c^\circ}{\sqrt{\pi}} + \frac{\partial w}{\partial t} - v\cos\beta\frac{\partial w}{\partial l} + v\sin\beta\right]$$

For cylindrical surfaces moving in the direction of their axes with velocity **v**:

$$\Delta p = -\left[\frac{(2-\varepsilon)}{\sqrt{\pi}}\rho_\infty c^\circ + \frac{\varepsilon\rho_\infty\sqrt{2\pi RT}}{4}\right]\left(\frac{\partial w}{\partial t} - v\frac{\partial w}{\partial l}\right) \tag{3.6}$$

$$\tau = -\frac{\varepsilon\rho_\infty v}{2}\left[\frac{c^\circ}{\sqrt{\pi}} + \frac{\partial w}{\partial t} - v\frac{\partial w}{\partial l}\right] \tag{3.7}$$

where the polar coordinates are the meridional angle ϕ and the distance l along the generators.

Expressions (3.6) and (3.7) are identical with those for the flat plate. Similar derivations can be carried out for still other surfaces which satisfy conditions (2.2).

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